OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes

* 4 7 2 5 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Evaluate $\sum_{r=101}^{250} r^3$. **[3]**

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the 2×2 identity matrix. Find the values of the constants a and b for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. **[4]**

3 The complex numbers z and w are given by $z = 5 - 2\text{i}$ and $w = 3 + 7\text{i}$. Giving your answers in the form $x + \text{i}y$ and showing clearly how you obtain them, find

(i) $4z - 3w$, **[2]**

(ii) z^*w. **[2]**

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \dfrac{1}{p} + \dfrac{1}{q}$. **[4]**

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[2]**

6 The complex number $3 - 3\text{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, **[3]**

(b) $\arg(z - a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geqslant 3\sqrt{2} \quad \text{and} \quad 0 \leqslant \arg(z - a) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

7 **(i)** Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^4 - r^4\} = (n+1)^4 - 1.$$ **[2]**

(ii) Show that $(r+1)^4 - r^4 \equiv 4r^3 + 6r^2 + 4r + 1$. **[2]**

(iii) Hence show that

$$4\sum_{r=1}^{n} r^3 = n^2(n+1)^2.$$ **[6]**

8 The matrix $\mathbf{C}$ is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.

(i) Draw a diagram showing the image of the unit square under the transformation represented by $\mathbf{C}$. **[3]**

The transformation represented by $\mathbf{C}$ is equivalent to a transformation S followed by another transformation T.

(ii) Given that S is a shear with the y-axis invariant in which the image of the point $(1, 1)$ is $(1, 2)$, write down the matrix that represents S. **[2]**

(iii) Find the matrix that represents transformation T and describe fully the transformation T. **[6]**

9 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{A}$. **[3]**

(ii) Hence find the values of a for which $\mathbf{A}$ is singular. **[3]**

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + y + z &= 2a, \\ x + ay + z &= -1, \\ x + y + 2z &= -1, \end{aligned}$$

have any solutions when

(a) $a = 0$,

(b) $a = 1$.

[4]

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

(i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 27$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**